Improving Efficiency in Stratified Audit Sampling via Bayesian Hierarchical Modeling

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Joint work with Lotte Mensink, Jacques de Swart, Eric-Jan Wagenmakers, and Ruud Wetzels

Stratified audit sampling is often used in practice but current statistical methodology has a limitation

Current landscape:

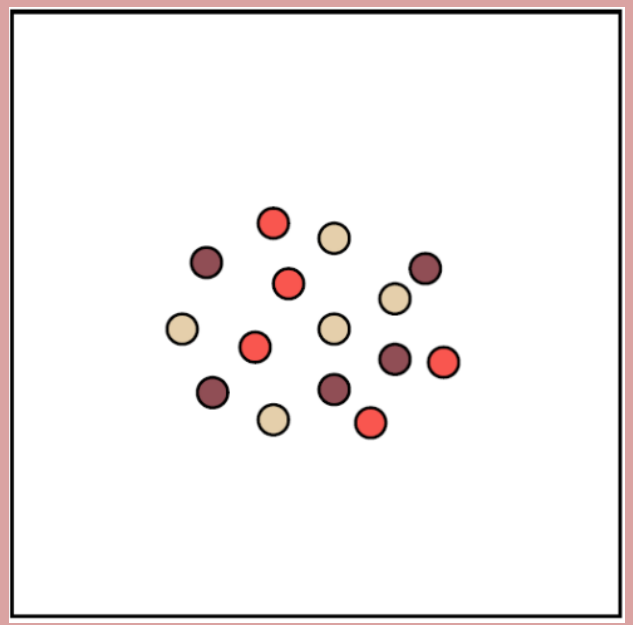
- Continuous demand for efficiency
- Sampling remains integral part of auditing
  - Full-population testing often difficult
- Statistical audit sampling is efficient
  - Quantify sampling risk = optimal sample size
- Stratification further increases efficiency
  - More accurate estimates = smaller samples

Limitation:

- Current stratified evaluation methods are inherently suboptimal
  - Do not consider similarities between strata
  - Testing too much samples

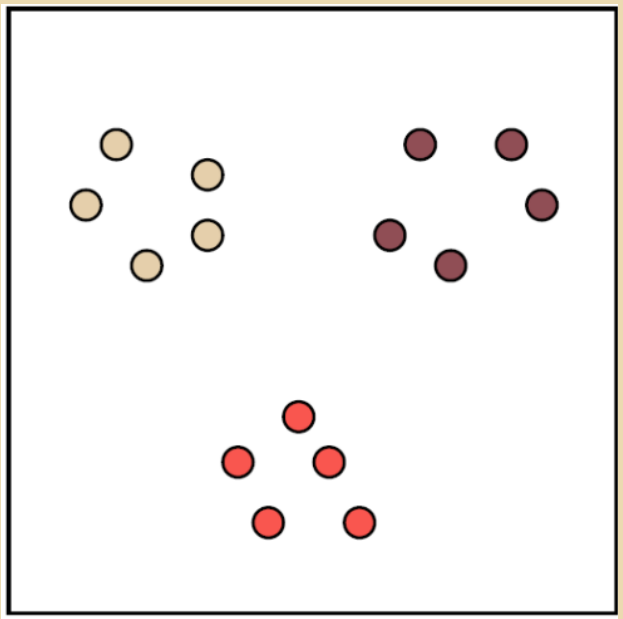
There are 3 ways to evaluate a stratified audit sample, which one is most suitable depends on the population at hand

Strata are identical to each other



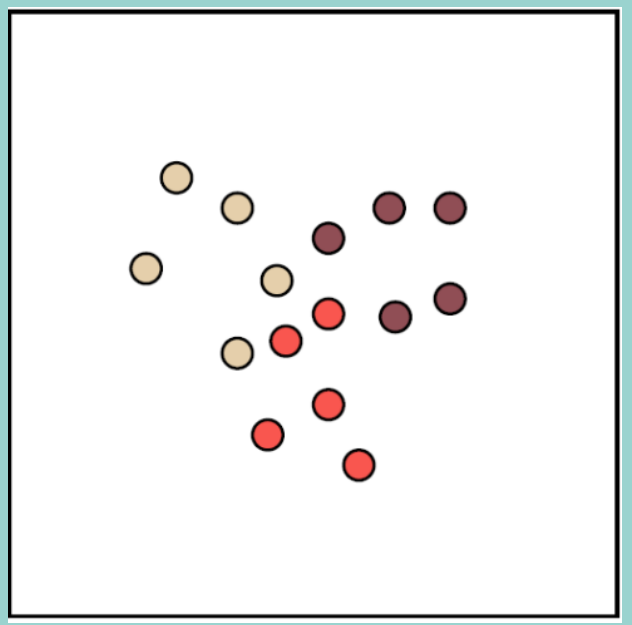
Aggregated approach

Strata are independent of each other



Independent approach

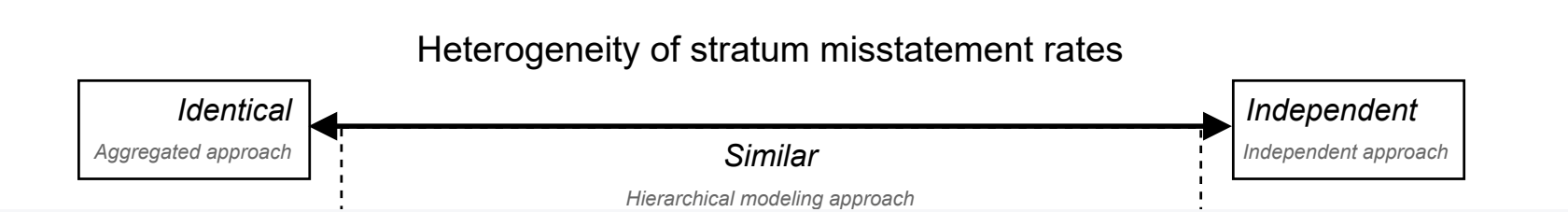
Strata are similar to each other



Hierarchical approach

When statistically analyzing a stratified audit sample, the hierarchical modeling approach is often the most suitable one

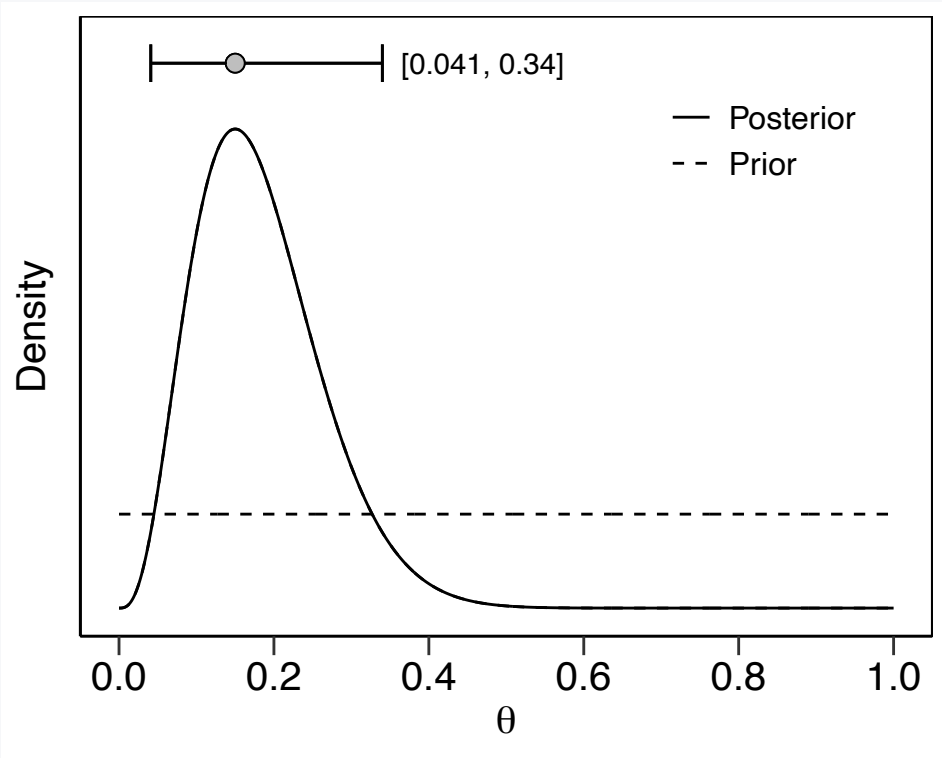
- Misstatement rates are often **not identical** across strata
  - E.g., branches of an auditee can have different personnel
- Misstatement rates are often **not independent** between strata
  - E.g., branches of an auditee belong to the same auditee
- More realistic that misstatement rates are **similar** to some degree



A statistical philosophy that aligns well with the hierarchical modeling approach and has practical advantages is Bayesian statistics

- Transparent incorporation and updating of pre-existing information
- Increases efficiency
- Ideally suited for hierarchical modeling

n = 20, k = 3



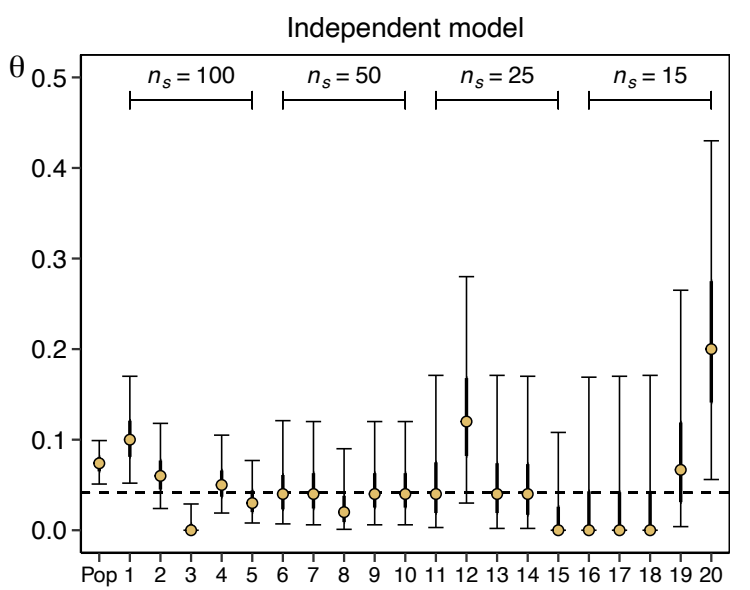
In this example, a group auditor must form an opinion on the group financial statements but also wants to form an opinion on the components

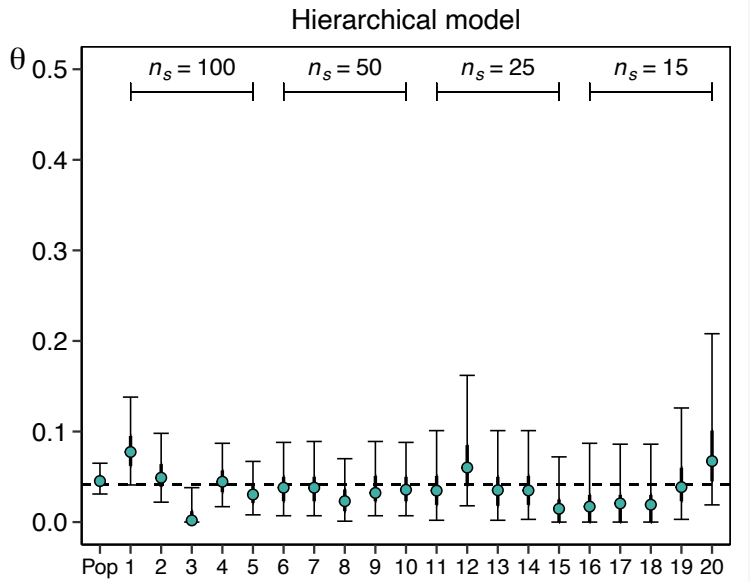
- Auditee is a furniture retailer with 20 branches
  - Centralized purchasing system
  - Individual inventory clerks
- The group auditor decides to stratify the population based on the 20 branches
  - Auditors check goods received notes

Branch (s)	Items (N_s)	Samples (n_s)	Misstatements (k_s)
1	5,000	100	10
2	5,000	100	6
3	5,000	100	0
4	5,000	100	5
5	5,000	100	3
6	5,000	50	3
7	5,000	50	3
8	5,000	50	1
9	5,000	50	2
10	5,000	50	2
11	10,000	25	1
12	10,000	25	3
13	10,000	25	1
14	10,000	25	1
15	10,000	25	0
16	10,000	15	0
17	10,000	15	0
18	10,000	15	0
19	10,000	15	1
20	4,000	15	3
Total	144,000	950	43

In this example, the hierarchical model Improves accuracy compared to the independent model

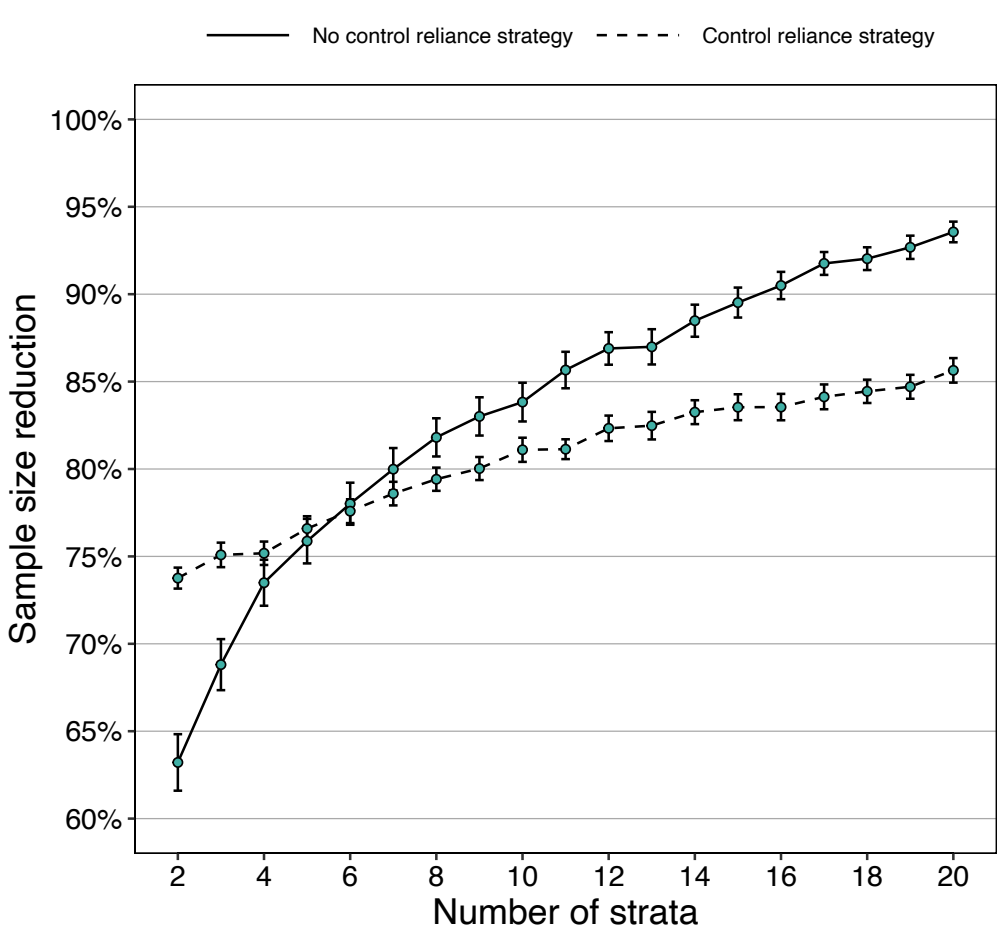
- Hierarchical model shrinks stratum estimates to the grand mean (dashed line) due to sharing info
- Degree of shrinkage partly depends on sample size *n*
  - e.g., stratum 1 (low) versus stratum 20 (high)
- Stratum estimates are **38%** less uncertain
- Population estimate (Pop) is **30%** less uncertain





Hierarchical modeling reduces the required sample size compared to the independent approach while being risk-free

- The hierarchical modeling approach is just as effective as the independent approach
- The hierarchical model has higher power at equivalent sample sizes
- Practical benefit: Sample size reduction
  - Ranges from **63%** with 2 strata up to **93%** with 20 strata
  - Control reliance strategy can be applied via the prior distributions



Conclusion and discussion

- Hierarchical modeling is often most suitable approach for stratified audit samples
- Bayesian hierarchical modeling does not compromise audit effectiveness
- Bayesian hierarchical modeling increases power and reduces required sample sizes
- Easy to apply via the open-source software JASP for Audit (Derks et al., 2021)
- ***What is the reason that auditors at the moment don't use these methods?***

References

- Derks, K., L. Mensink, J. de Swart, E.-J. Wagenmakers, and R. Wetzels. 2024. Improving efficiency in stratified audit sampling via Bayesian hierarchical modeling. *PsyArXiv.* https://doi.org/10.31234/osf.io/tgq5z

- Derks, K. J. de Swart, E.-J. Wagenmakers, J. Wille, and R. Wetzels. 2021. JASP for Audit: Bayesian tools for the auditing practice. *Journal of Open Source Software* 6 (68): 2733. https://doi.org/10.21105/joss.02733

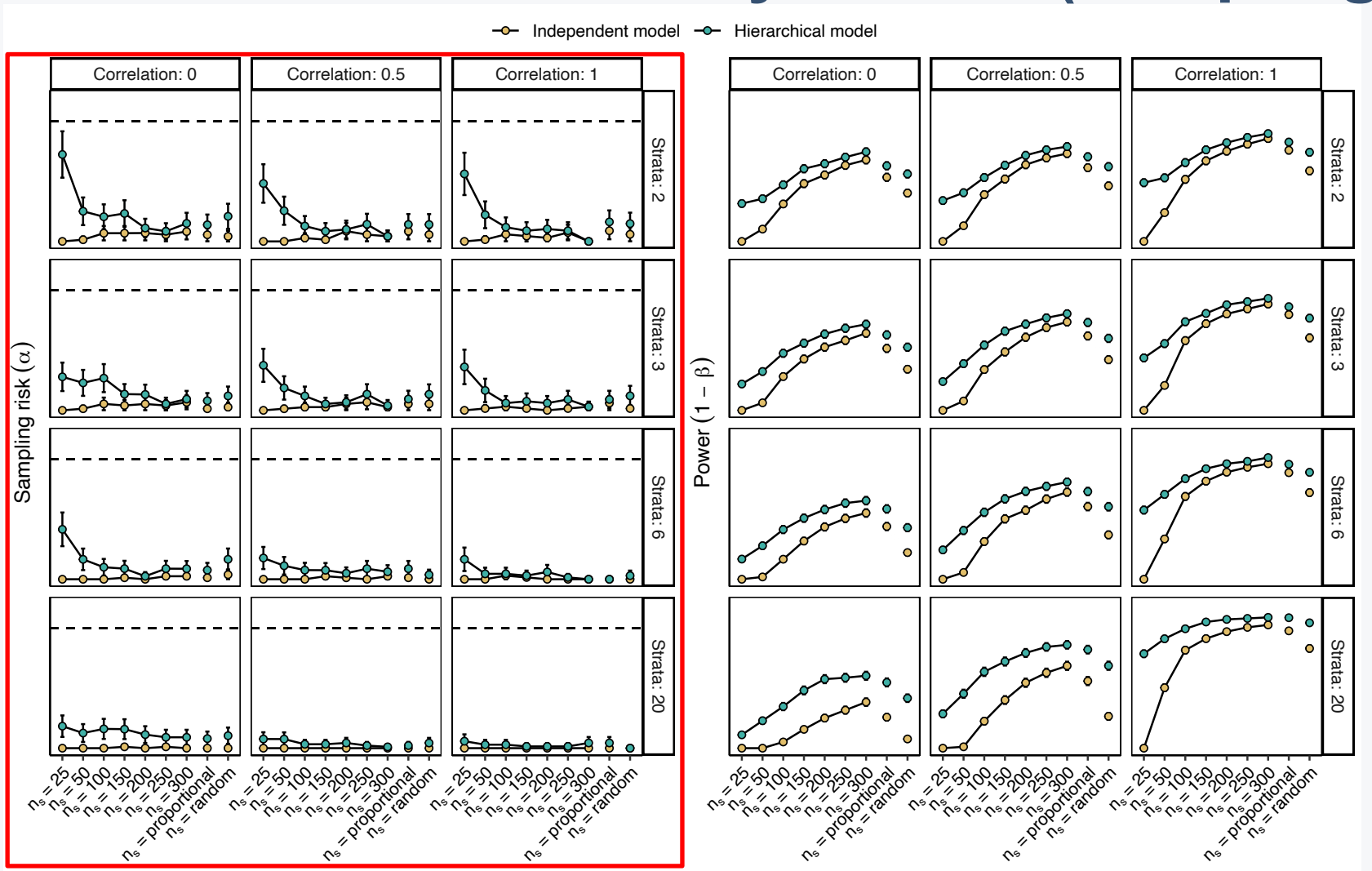
- Durney, M., R. J. Elder, and S. M. Glover. 2014. Field data on accounting error rates and audit sampling. *Auditing: A Journal of Practice & Theory* 33 (2): 79–110. https://doi.org/10.2308/ajpt-50669

Appendix A: Simulation study design

- Realistic audit conditions
  - Strata: $S \in \{2, 3, 6, 20\}$
  - Correlation: $r \in \{0, 0.5, 1\}$
  - 95% of misstatement rates < 0.1 (Durney, Elder, and Glover, 2014, Table 3, Panel A)
  - $n_s \in \{25, 50, 100, 150, 200, 250, 300,$ proportional = $0.3N_s,$ random$\}$
- Outcome measures for H_0: θ (misstatement) $\geq \theta_{max}$ (tolerable misstatement)
  - Effectiveness: α (sampling) risk
  - Efficiency: $1 - \beta$ (power)

	H_0 is true	H_0 is false
Reject H_0	Type-I error (α risk)	Correct decision ($1 - \beta$, Power)
Do not reject H_0	Correct decision ($1 - \alpha$)	Type-II error (β risk)

Appendix B: Simulation study results (sampling risk)



Appendix C: Simulation study results (power)